

**PREDICTED NEUTRINO FLUXES FOR IDEAL NARROW-BAND SYSTEMS
(USING THE MULTIPERIPHERAL MODEL OF CLIFFORD RISK)**

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I. INTRODUCTION

The purpose of this report is to summarize the "narrow-band" neutrino fluxes together with the expected neutrino-interaction rates in the hydrogen-filled 30 m^3 NAL bubble chamber. The multiperipheral model of Clifford Risk is used because of its success in fitting the recent ISR data at equivalent proton energies of 500 and 1100 GeV as well as data at 30 GeV.

Details of this model showing the agreement at 30 GeV have been published.¹ The agreement with the ISR data (soon to be published²) is summarized in Figs. 1a, b, and c. We can expect that this model will make predictions at 200, 300, and 400 GeV that will be within about 30% of the actual meson yields (since interpolation rather than extrapolation is now involved). No attempt has been made to predict K^- yields; therefore, this summary will be for neutrinos from π^\pm and K^\pm . Furthermore, the meson production cross-section measurements are always made with very thin targets whereas the neutrino target will be one mean-free-path thick. The effects of multiple interactions of the protons and the subsequent meson absorption in thick targets have been



studied previously.³ The way in which a thick target distorts the thin target meson momentum spectrum is displayed in Fig. 2 (reproduced from TM-218, Fig. 4b) as $\frac{dn}{dq} / \left(\frac{dn}{dq} \right)$ undistorted very thin target vs q for various thickness targets. Here, q is a dimensionless quantity proportional to the meson momentum,

$$q \equiv p / (T_0 = 0.305 E_0^{3/4}),$$

and E_0 is the incident proton energy. The ratio,
 $\frac{dn}{dq} / \left(\frac{dn}{dq} \right)$, here defined as a "distortion factor" is plotted vs meson momentum p in Fig. 3 for a one mean-free-path target ($\ell/\lambda = 1$) for three different incident proton energies 200, 300, and 400 GeV. To convert the yields per interacting proton (from an infinitely thin target) summarized in the remainder of this memo to yields per incident proton one simply multiplies by this distortion factor.

II. IDEAL NARROW-BAND BEAM⁴

The neutrino fluxes are calculated on the assumption that an ideal meson focusing device exists that will take all mesons of momentum between p and $p + \Delta p$, and all production angles between θ_1 and θ_0 and bend them so they would pass through the center of the bubble chamber located 1400 meters away. A 1000-meter thick shield located 400 meters downstream of the target prevents them from reaching the center of the chamber. Some of the mesons decay in the 400 meter drift space. Those neutrinos from the dominant decay mode, $\mu + \nu$, that fall within

a narrow enough cone in the meson rest frame will pass through the bubble chamber [at a distance $\leq R$ (meters) from the axis of the neutrino beam]. The energy of the neutrino that strikes a distance R from the axis is given by

$$E_{\nu} = 2\eta q' / \{ 1 + [\eta R / (D + S)]^2 \}, \quad (1)$$

where q' is the momentum of the neutrino in the meson rest frame and η is the meson momentum divided by its rest mass.

The fraction Y of the meson decays that have neutrinos passing through a detector of radius R is

$$Y = fr^2 e^{-x} \int_{x_0}^x \frac{e^y dy}{r^2 + y^2}, \quad (2)$$

where

f = two-body leptonic branching ratio

r = $R/c\tau$

τ = meson mean life

c = velocity of light

y = $L/\eta c\tau$

x = $(D + S)/\eta c\tau$

L = distance from meson decay point to detector

D = length of decay space

S = distance from beginning of shield to detector

x_0 = $S/\eta c\tau$.

The full fractional neutrino energy spread is given by

$$\Delta E_\nu / E_{\nu \max} \equiv 1 - \frac{E_{\nu \min}}{2\eta q'} = 1 - \{ 1 / [1 + (\eta R/S)^2] \}. \quad (3)$$

The variation of Y and $\Delta E_\nu / E_{\nu \max}$ with pion (kaon) momentum are given in Fig. 4a and b for detector radii of 0.5, 1.0, 1.5, 2.0, and 2.5 meters. Here $D = 400$ m and $S = 1000$ m. Figure 5 gives the pion and kaon neutrino energy ($E_{\nu \max}$) as a function of meson momentum.

III. MESON YIELDS

It is arbitrarily assumed that the focusing system will have a full width at half maximum acceptance of $\Delta p/p = 0.1$ for any momentum setting. The yields can be scaled to correspond to the actual device. Figures 6a, b, c, 7a, b, c, and 8a, b, c, show the number of mesons emitted between 0 and θ_0 (milliradians) for 200 GeV, 300 GeV, and 400 GeV interacting protons respectively, where "a" = π^+ , "b" = π^- , "c" = K^+ . Here θ_0 takes on the values 1 through 10 milliradians in one unit steps. By subtracting the curve for θ_1 from that for θ_0 one can obtain the meson yields for any arbitrary ideal focusing device.

IV. NEUTRINO YIELDS

For the remainder of this summary we give the yields for a detector radius of $R = 1$ meter. Values for other radii are available upon request. Figures 9a, b, c, 10a, b, c, and 11a, b, c give the number of neutrinos passing through a detector of 1 meter radius per 10^5 interacting protons of energy 200 GeV, 300 GeV, and 400 GeV respectively.

V. NEUTRINO INTERACTION RATES
IN NAL 30 m³ LIQUID HYDROGEN CHAMBER

Figures 12a, b, c, 13a, b, c, and 14a, b, c, give the number of neutrino interactions within 1 meter of the axis of the bubble chamber per 10^{19} interacting protons of energy 200 GeV, 300 GeV, and 400 GeV, respectively. The neutrino cross section is assumed to be $0.8E(\text{GeV})10^{-38}$ cm². To obtain the rates per incident proton on a one mean-free-path thick target one must multiply by the distortion factor of Fig. 2.

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⁴The concept of the narrow-band beam was first enunciated by Vincent Peterson. Since then variations upon it have appeared.
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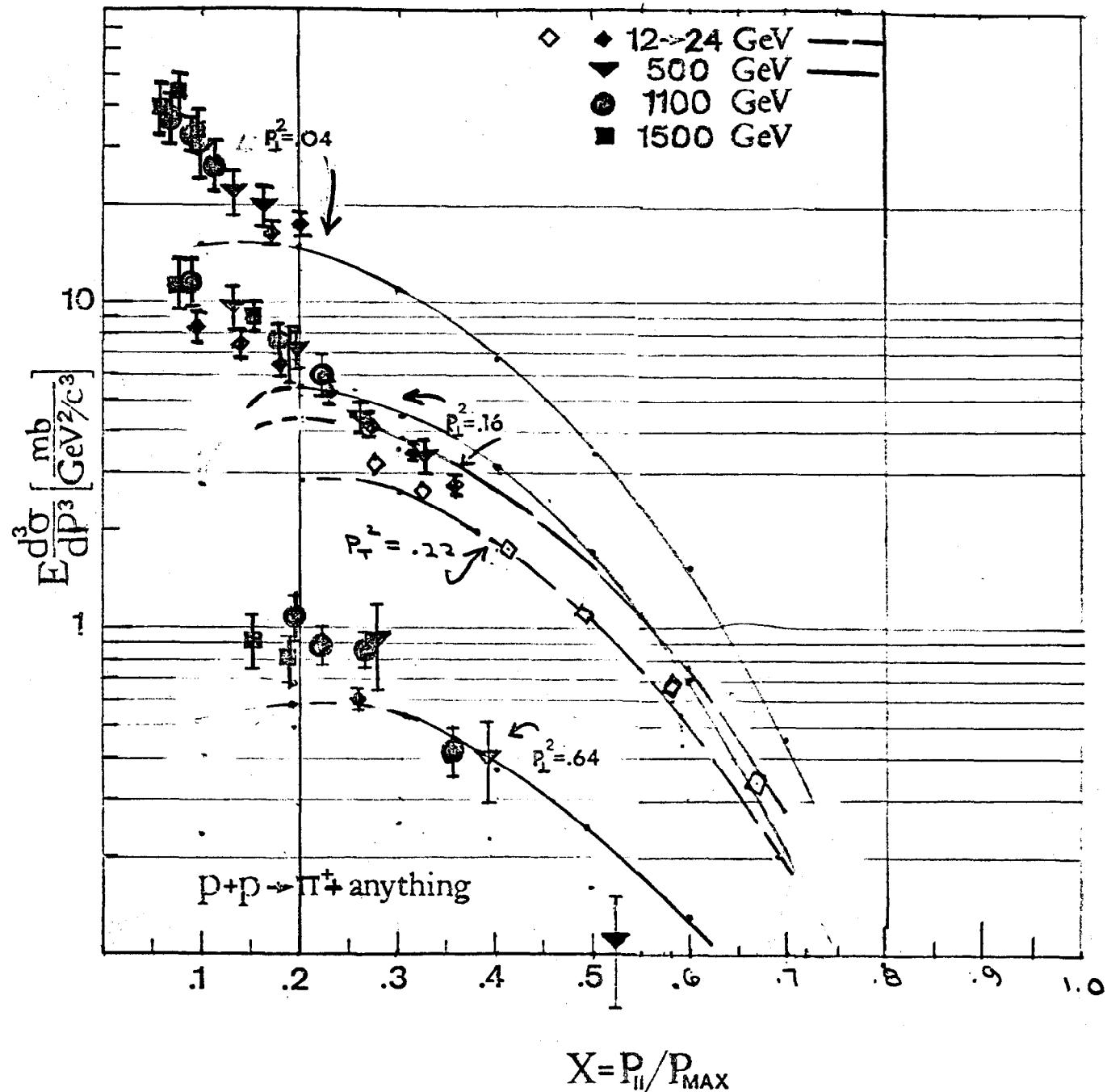
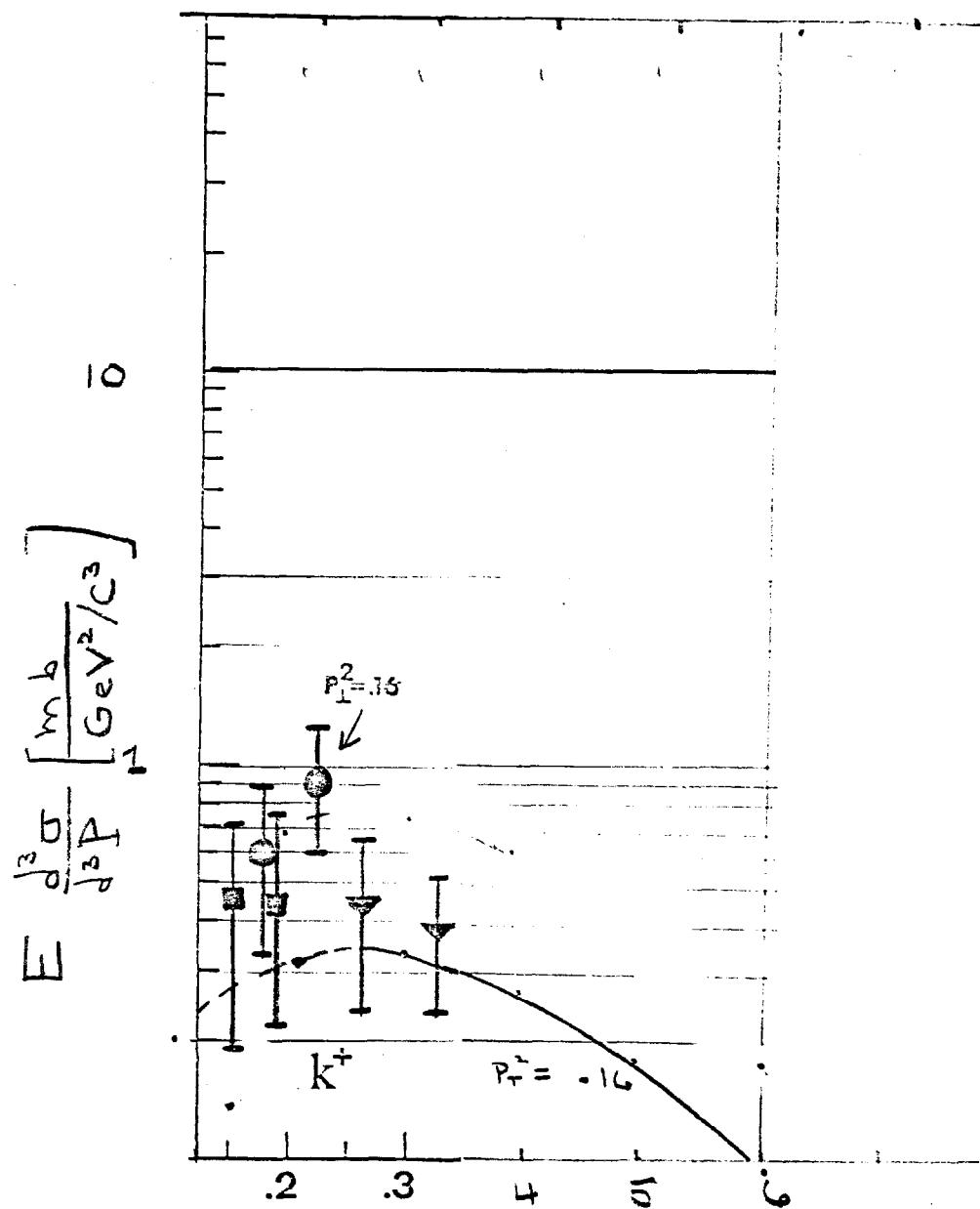


Fig. 1a



$$X = P_{||} / P_{MAX}$$

Fig. 1b

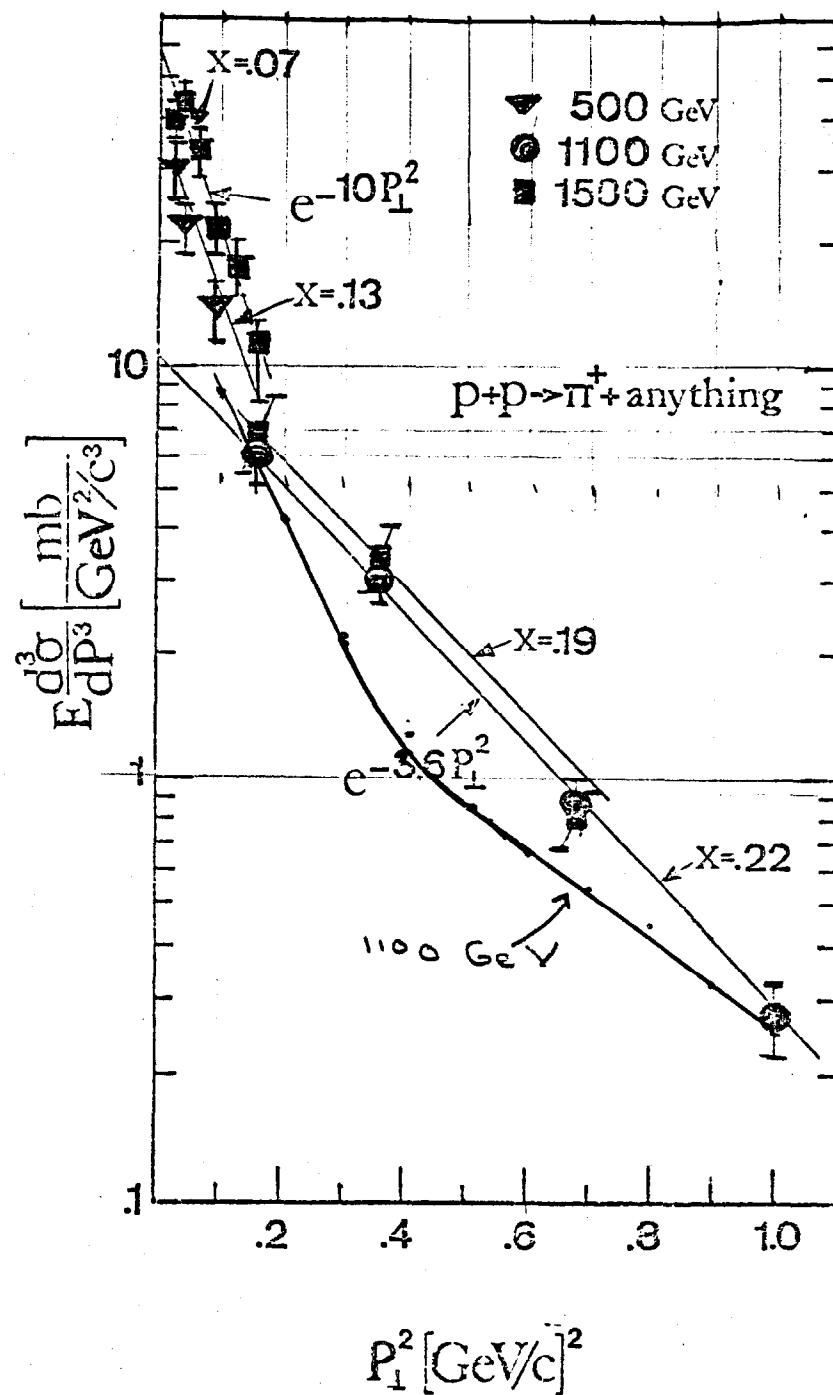


Fig. 1C

$$\frac{dn}{dq} / \left(\frac{dn}{dq} \right)_{\text{undistorted}} = \left(\frac{\lambda}{\ell} \right)^{-1} \frac{e^{\ell/\lambda b} - 1}{b} e^{-\ell/\lambda \pi}$$

very thin target

$$b \equiv \frac{e^{-3Kq/4} - \beta}{\beta^2} \left(1 - \frac{\lambda}{\lambda_\pi} \right)$$

$$\lambda_\pi = \lambda$$

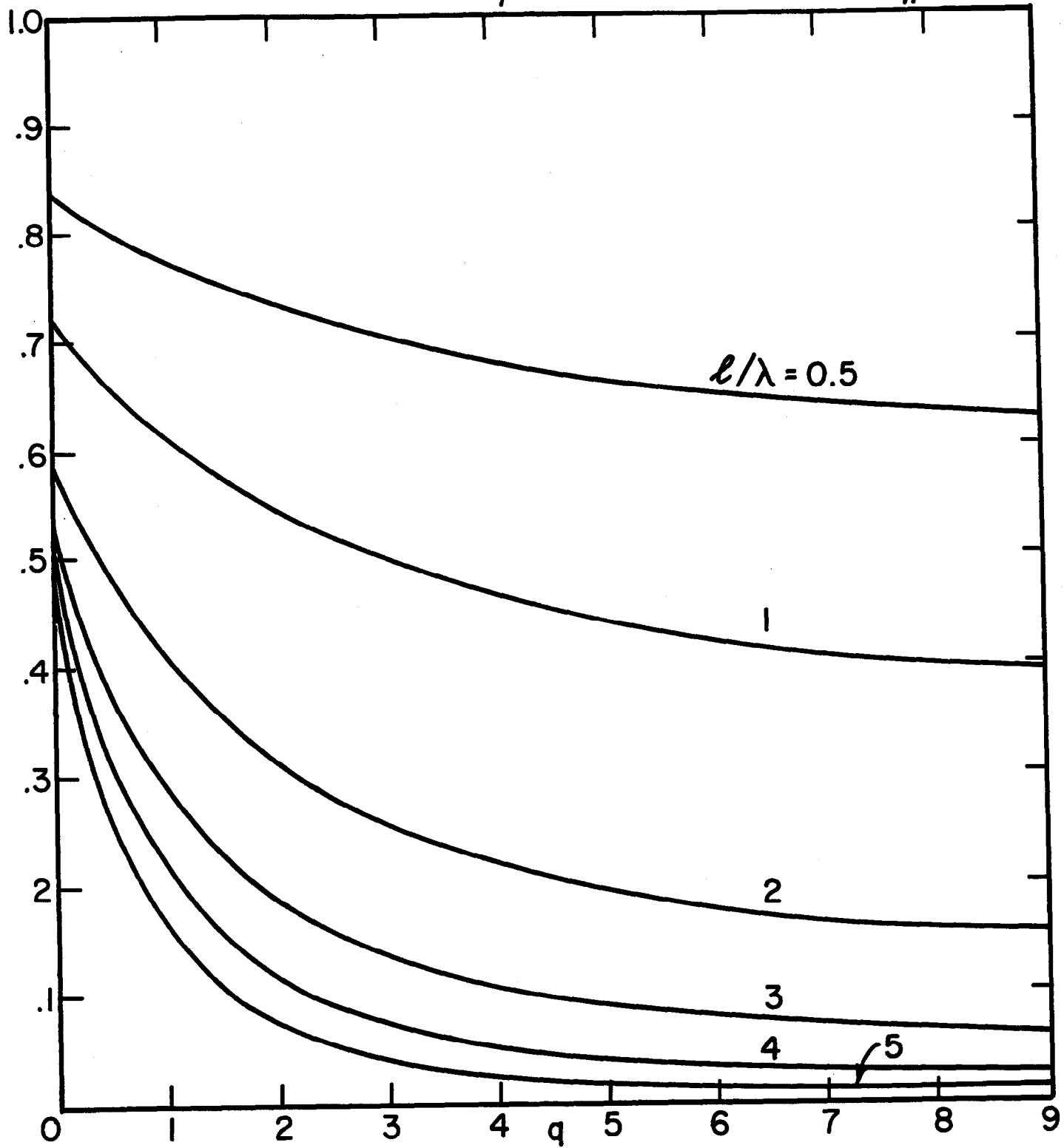
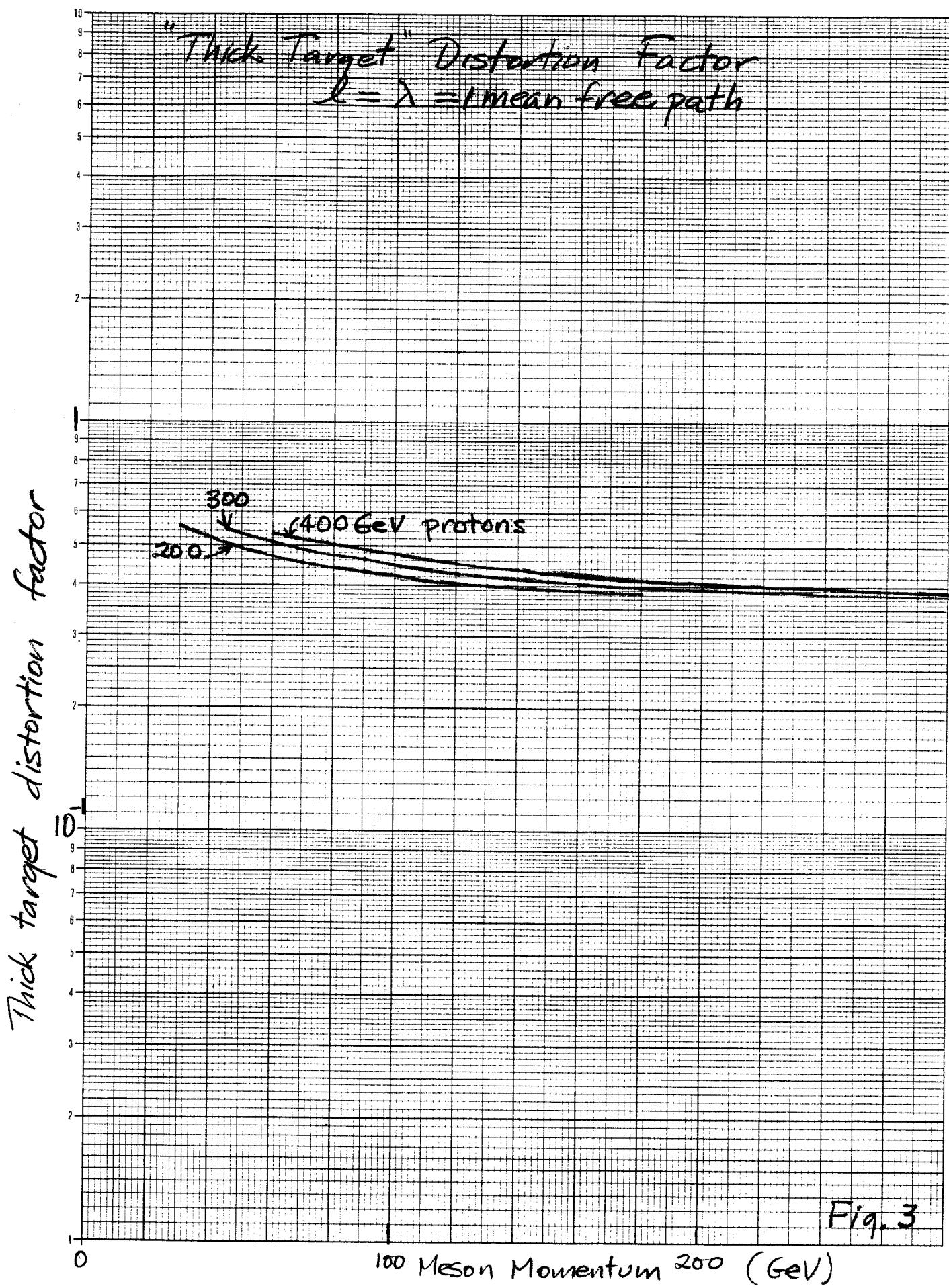
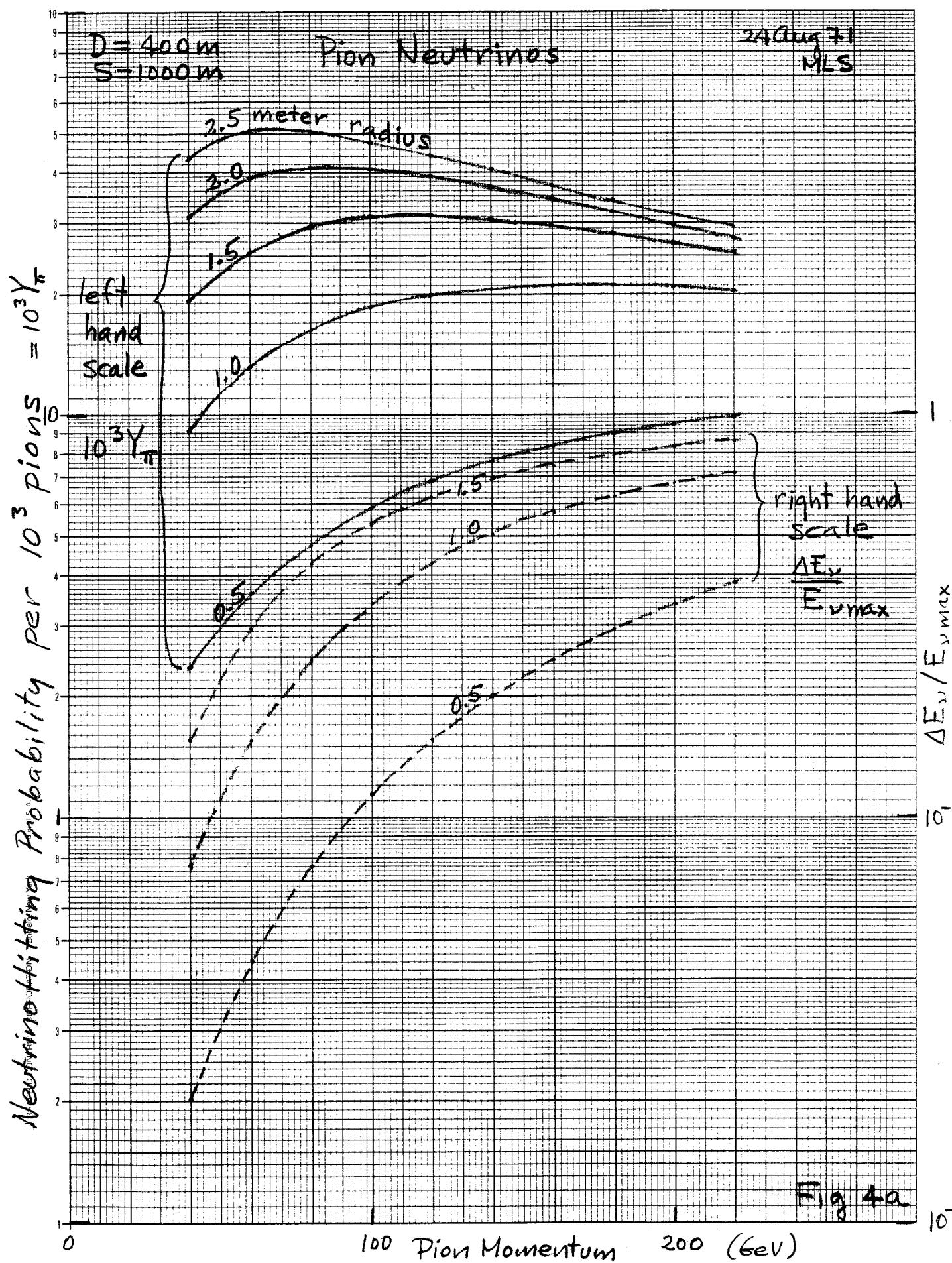
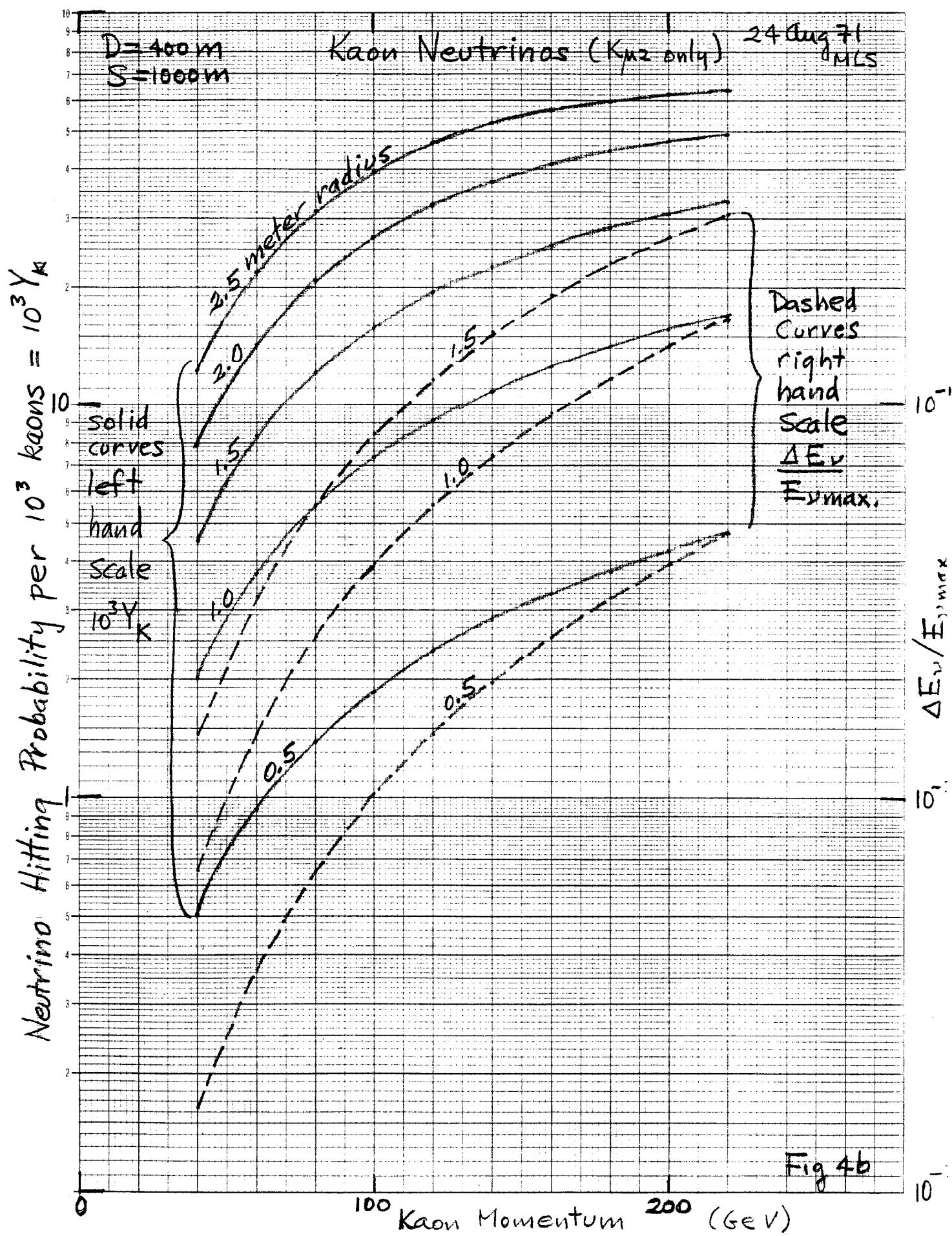
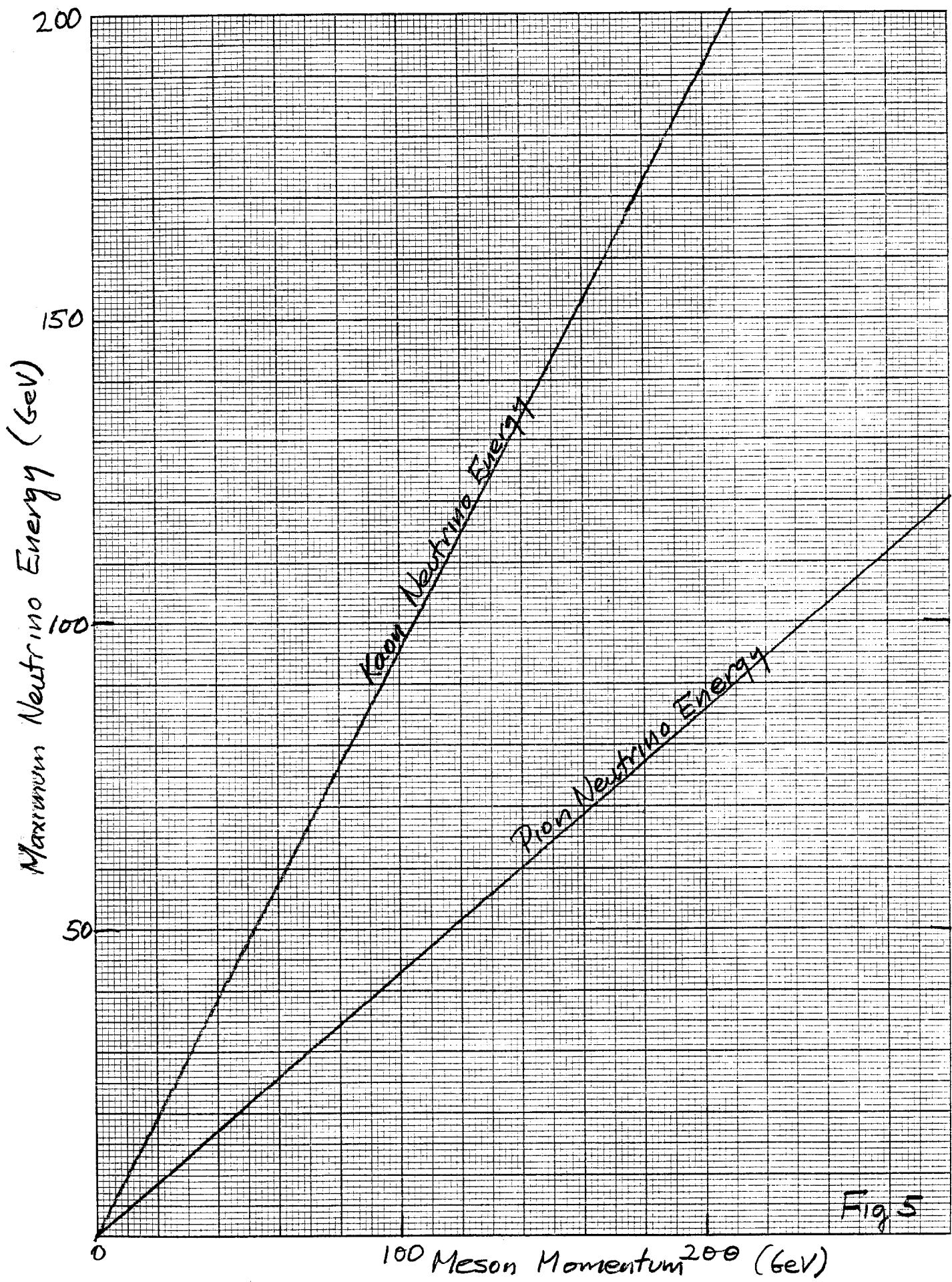


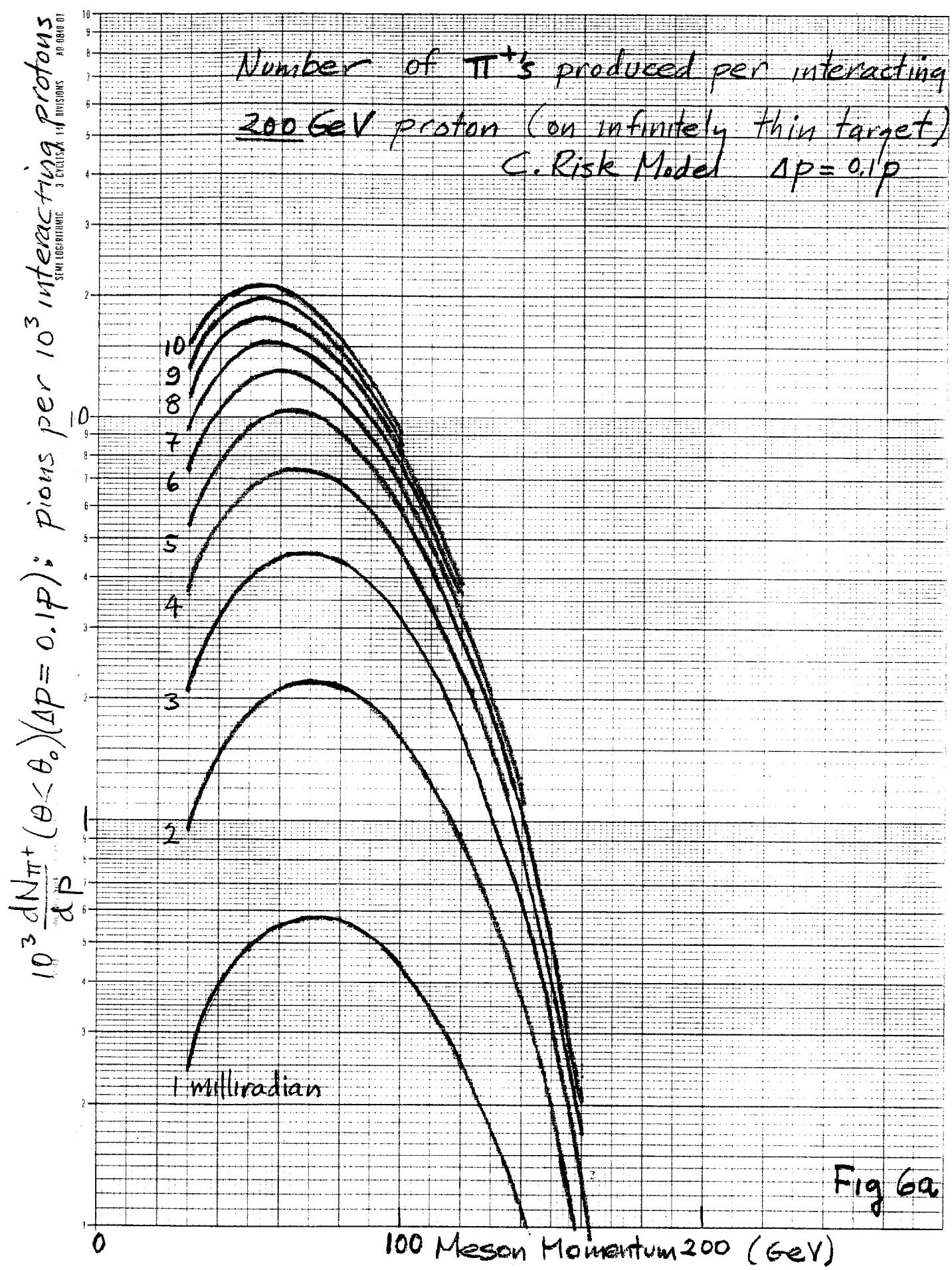
Fig. 2

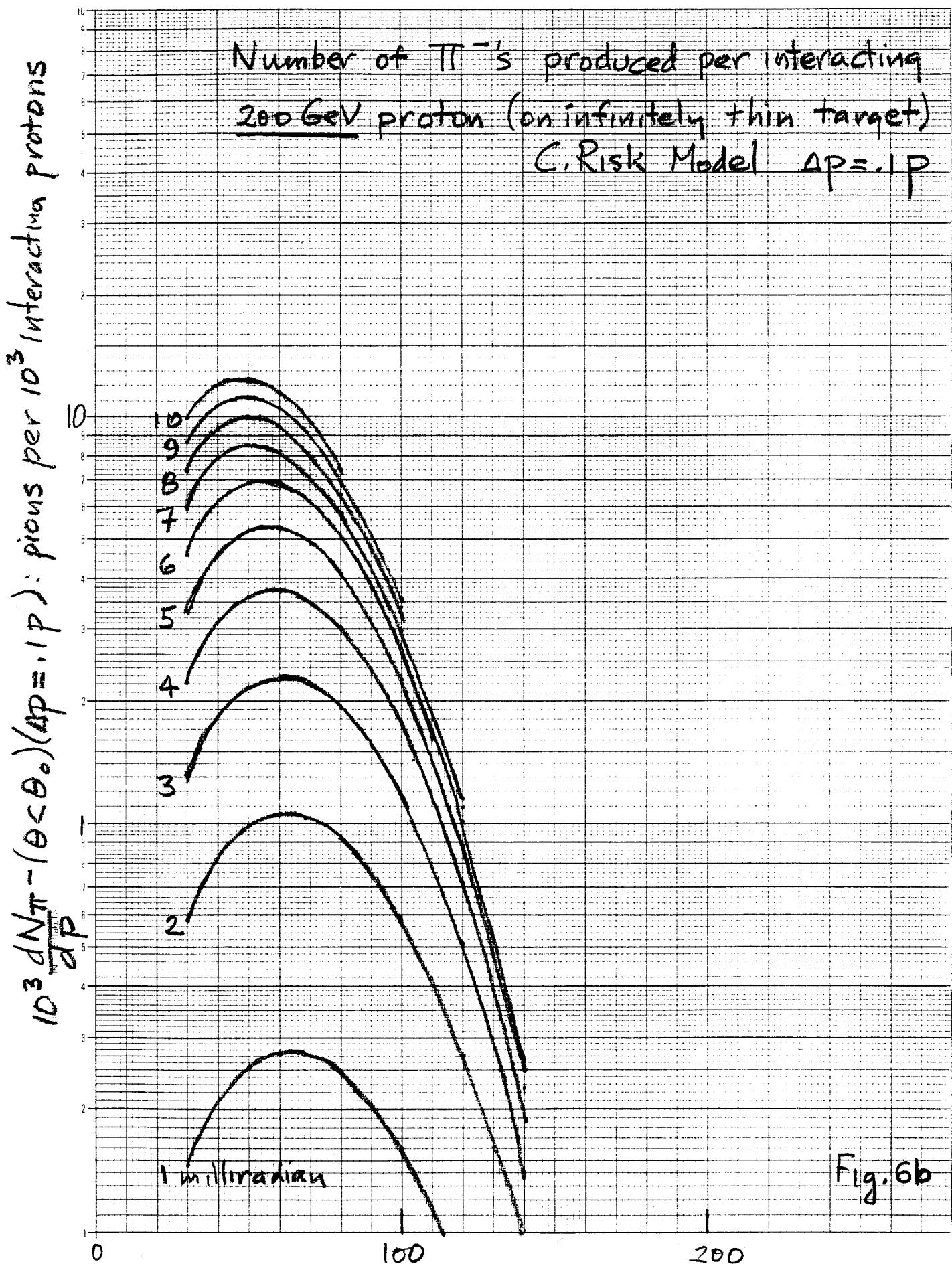


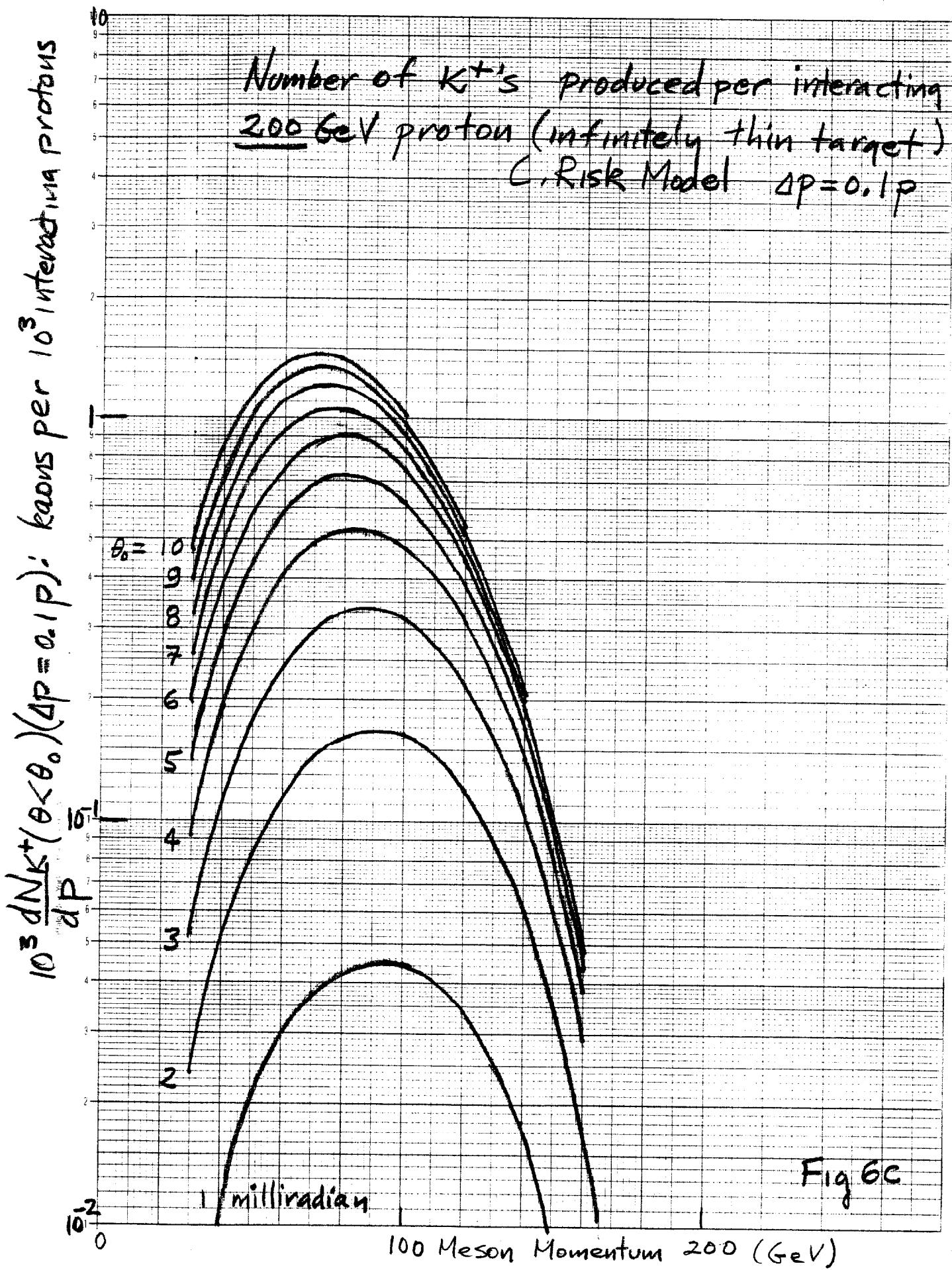


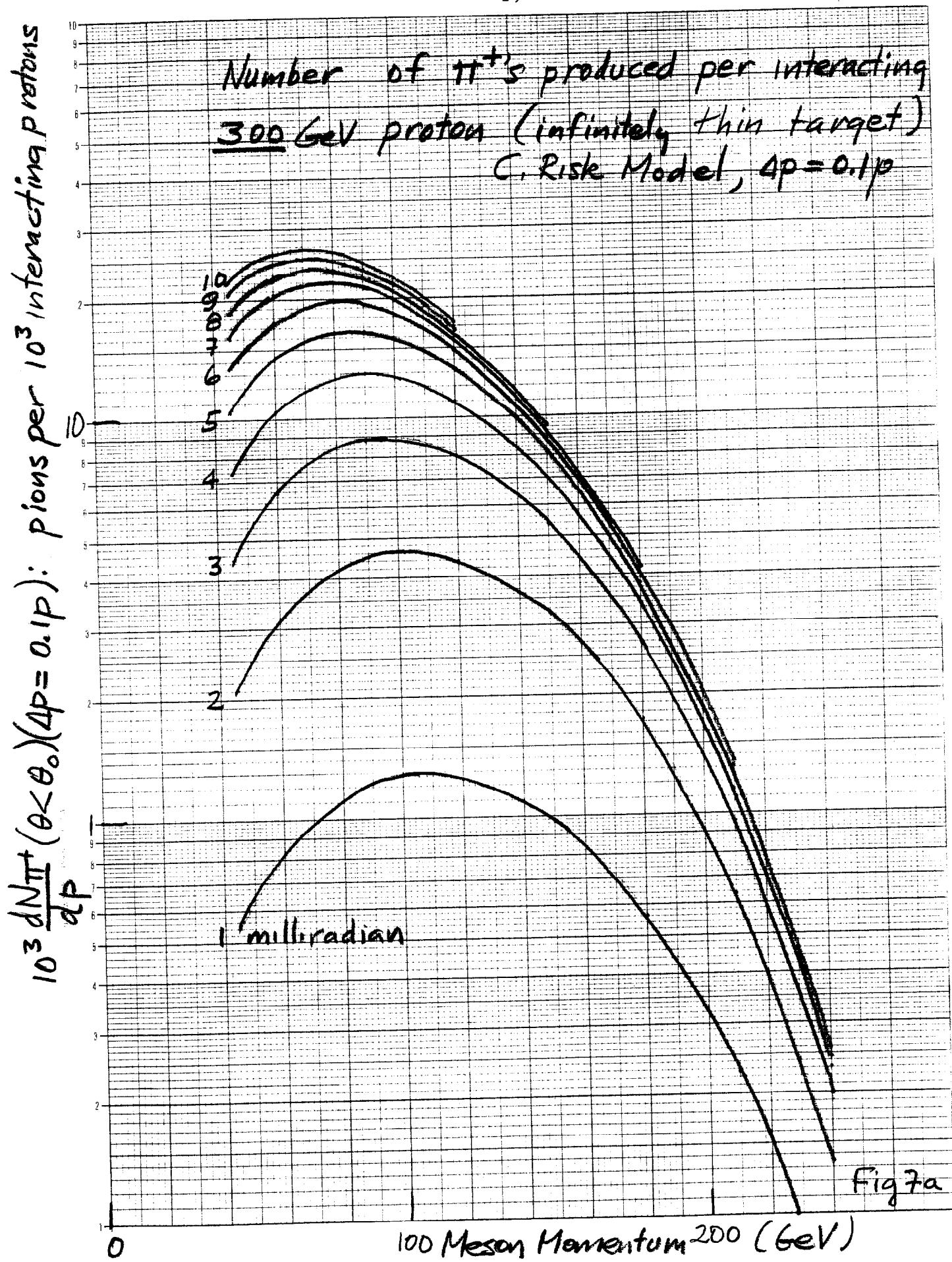


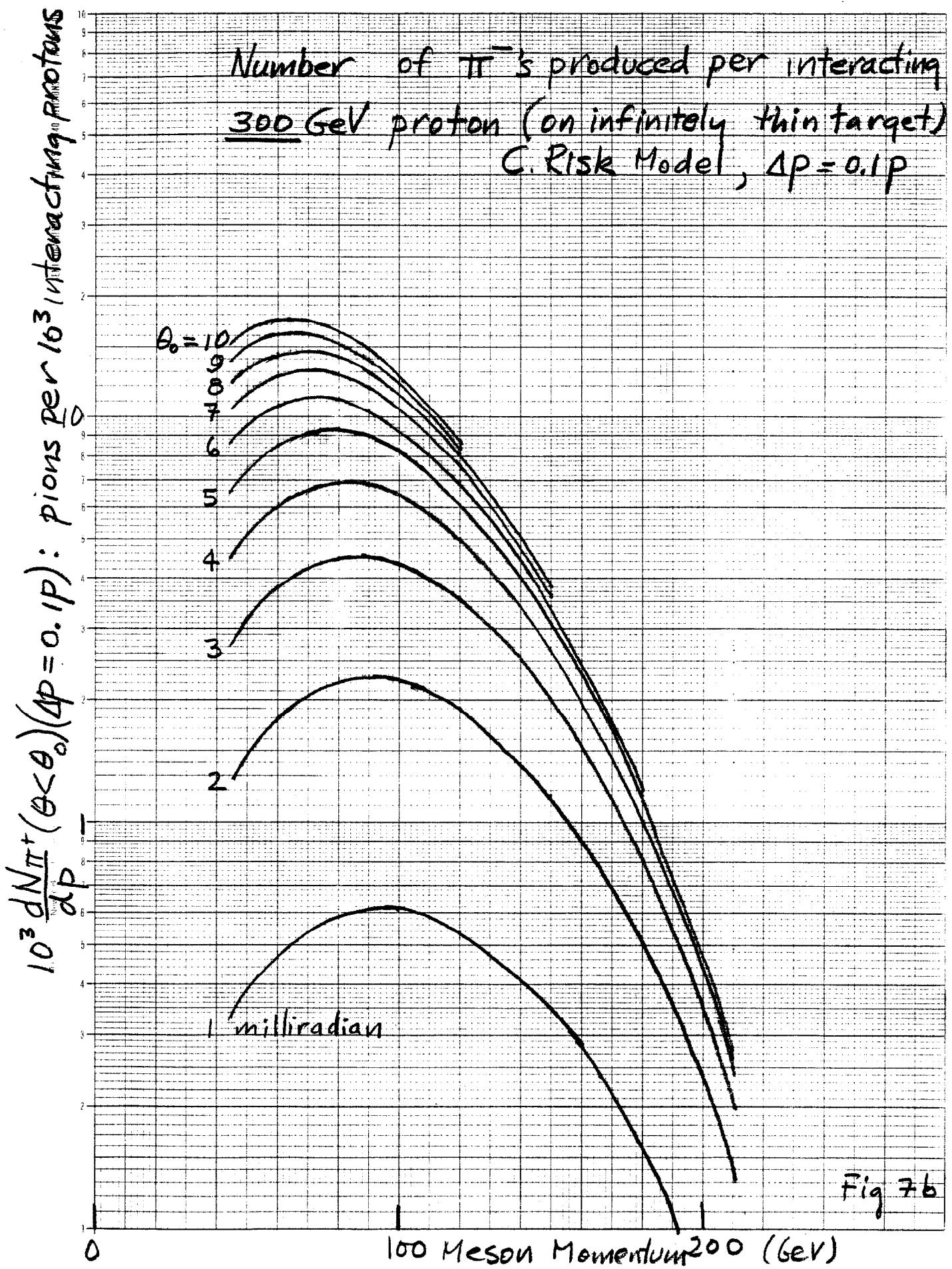


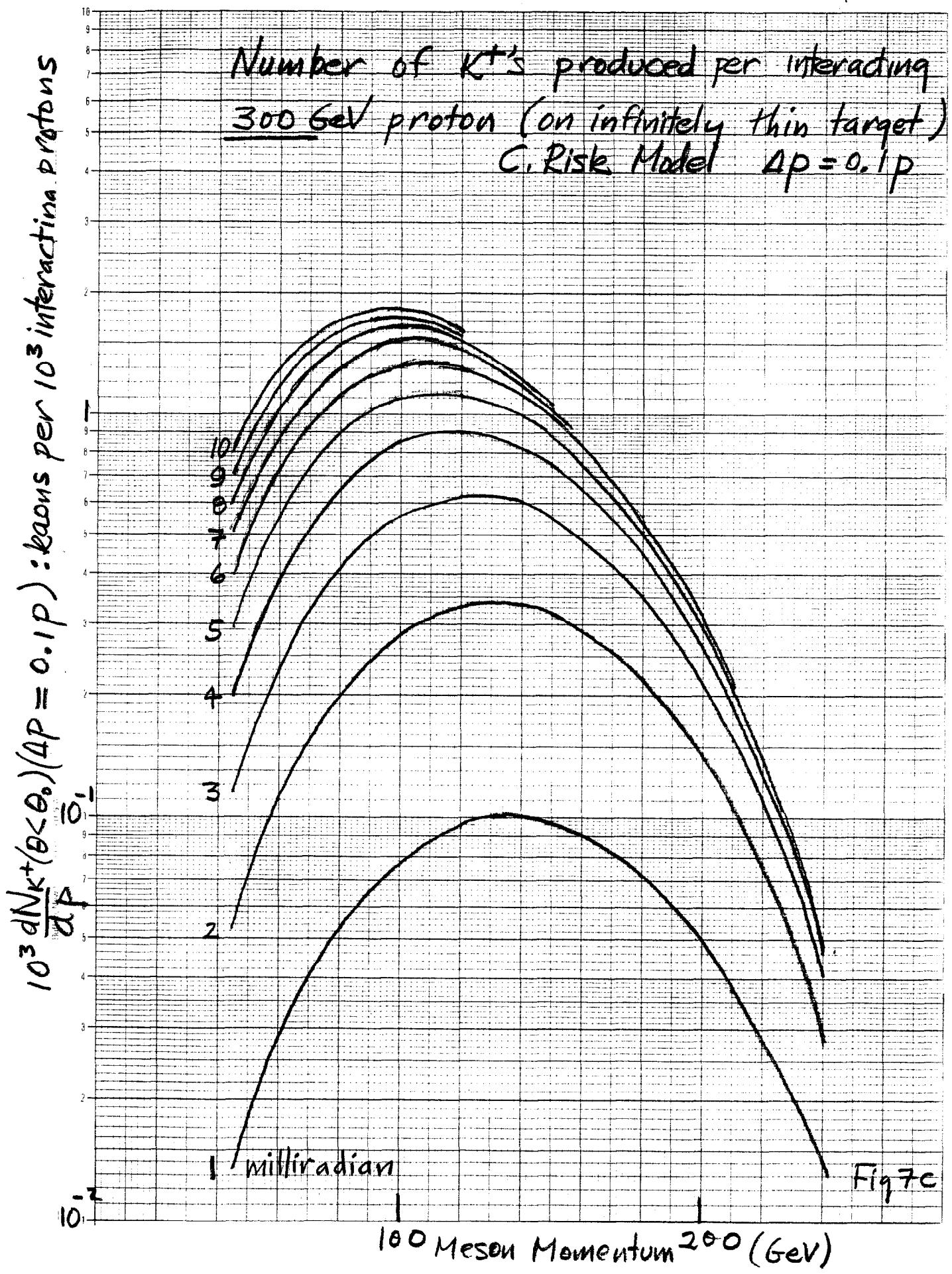




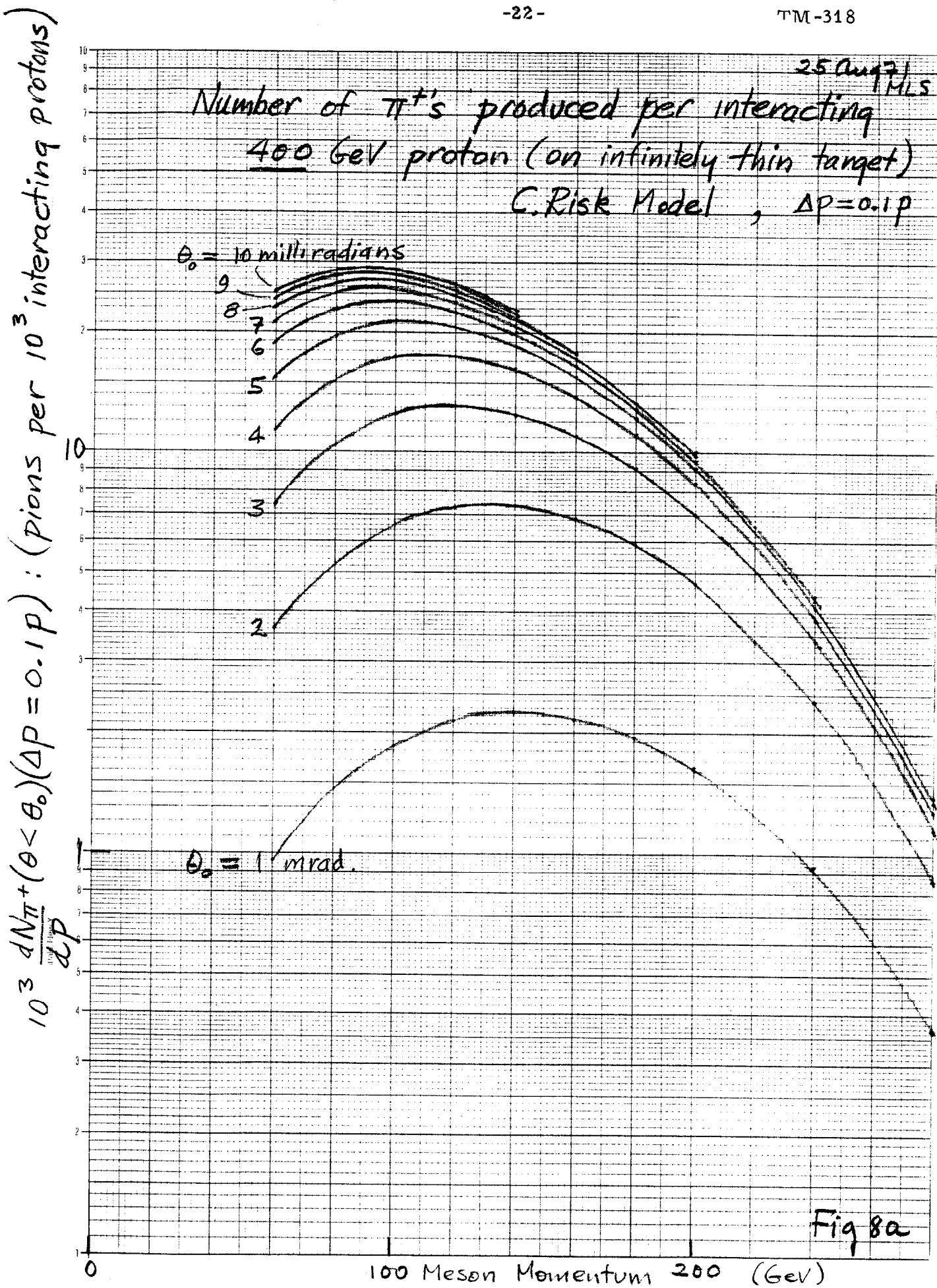








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